GEOMETRICAL FACTORS FOR PARTS OF AN INFINITE CYLINDRICAL SURFACE

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A method is described for calculating geometrical factors, allowing for attenuation of radiation by the medium, and computing radiative heat transfer between coaxial strips forming part of a cylinder.

In the solution of various problems of thermal and optical engineering by the zonal method, geometrical factors are required. The complexities of calculation of these factors frequently gives rise to crude simplification of the system. For example, in widely used cylindrical chambers, the surface is assumed to be a single zone, whereas the temperature often varies markedly around the chamber, and the surface in this case should be subdivided into a number of zones—coaxial strips, as shown in the figure, where 1, 2, ..., i, ... n are strip subscripts. In the computation it is required to determine n^2 geometrical factors for the various strips, and these may conveniently be reduced to tabular (matrix) form:

$$\begin{pmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1n} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \varphi_{n1} & \varphi_{n2} & \cdots & \varphi_{nn} \end{pmatrix}. \qquad (1)$$

We have developed a method of calculating the elements of the matrix (1) for the strips of a cylindrical surface.

For any system of zones we may construct matrix (1), but direct calculation of all n^2 factors from the well-known formulas is too complicated. The total number of operations may be reduced by choosing a minimum group of basic factors, calculated directly. The elements of matrix (1) are calculated from the basic factors using algebraic relations, the order of operations being as follows:

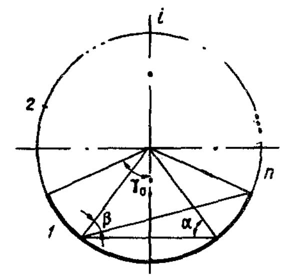
1. The matrix (1) is constructed.
2. The group of basic factors is chosen.
3. The algorithm for determining all the factors from the basic factors is completed.
4. The required values of the basic factors are tabulated.

We will carry out this plan for a system of strips constituting a cylinder.

We first examine more generally than usual, the case when the indicatrix of radiation of the surface elements is symmetrical and identical for all the elements, while the values of the attenuation coefficients vary throughout the volume, being symmetrical, however, about the axis. Then:

1) the reciprocal relation for our geometrical factors will be the same as for diffuse radiation, which is easy to find from the geometrical factor formula with a symmetrical indicatrix [1];

2) from the basic conventional meaning of geometrical factors it is evident that the local factor for a closed cylindrical surface ξ will be the same for all points and equal to the sum of the factors in any row of matrix (1).



Section showing the system of strips forming part of a cylindrical surface.

Analysis shows that it is most suitable to take as the basic geometrical factors the factors for the strips themselves φ_{mm}. Here the word "strip" has a wide meaning, i.e., the index m may refer to a combination of any number of adjacent strips or zones. Thus we require a table of φ_{mm}, where m combines the subscripts:

$$\begin{array}{lll} 1 & & \\ 1,\ 2 & 2 & \\ 1,\ 2,\ 3 & 2,\ 3 & \\ \cdots\cdots & \cdots\cdots & \cdots \\ 1,\ 2,\ 3 \ldots n-1 & 2,\ 3 \ldots n-1,\ \ldots, & n-1 \\ 1,\ 2,\ 3 \ldots n-1,\ n & 2,\ 3 \ldots n-1,\ n,\ \ldots, & n-1,\ n\ \ n. \end{array} \qquad (2)$$

The table of φ_{mm} must include the quantity $\xi = \varphi_{mm}$, when m is the sum of all the strips composing the closed shell.

All the diagonal elements of matrix (1) are taken from the table of φ_{mm}.

We examine any two adjacent strips: i and i + 1. Let the subscript j combine i and i + 1. According to the distributive property of fluxes,

$$F_j \varphi_{jj} = F_i (\varphi_{ii} + \varphi_{i,\,i+1}) + F_{i+1} (\varphi_{i+1,\,i} + \varphi_{i+1,\,i+1}). \quad (3)$$

From the reciprocal relation

$$F_i \varphi_{i,\,i+1} = F_{i+1} \varphi_{i+1,\,i}. \quad (4)$$

Since the values φ_{jj}, φ_{ii}, $\varphi_{i+1,i+1}$ are taken from the table of φ_{mm}, Eqs. (3) and (4) give us the factors $\varphi_{i,\,i+1}$ and $\varphi_{i+1,\,i}$, i.e., the factors for the adjacent strips—φ_{12}, φ_{21}, φ_{23}, φ_{32}, φ_{34} ..., etc.

We now examine any three adjacent strips: i, i + 1, i + 2. Let the index j combine as before strips i, i + 1,

Table 1
Geometrical Factors (the decimal point precedes the numbers shown)

Δ	φ_{mm} at γ_0, degree					
	15	30	45	60	75	90
0	01137	04508	09963	17285	26189	36321
0.01	01135	04493	09916	17181	26000	36026
0.05	01127	04435	09730	16770	25263	34876
0.1	01118	04364	09505	16275	24382	33507
0.2	01100	04227	09076	15344	22740	30974
0.3	01082	04096	08672	14483	21240	28683
0.4	01065	03979	08293	13686	19866	26605
0.6	01031	03735	07597	12253	17443	22987
0.8	00999	03516	06976	11099	15385	19967
1.0	00968	03315	06429	09922	13625	17427
1.2	00938	03129	05929	08970	12113	15279
1.5	00896	02874	05261	07750	10222	12644
2.0	00831	02508	04359	06153	07833	09408
2.5	00772	02199	03648	04958	06119	07163
3	00719	01938	03082	04249	04865	05579
5	00547	01224	01700	02022	02255	02435
7	00425	00821	01041	01168	01253	01316
10	00301	00495	00576	00616	00643	00665

while the index m combines all three strips. As in (3) and (4), we write

$$F_m\varphi_{mm} = F_j(\varphi_{jj} + \varphi_{j,\,i+2}) + F_{i+2}(\varphi_{i+2,\,j} + \varphi_{i+2,\,i+2}), \quad (5)$$

$$F_j\varphi_{j,\,i+2} = F_{i+2}\varphi_{i+2,\,j}. \quad (6)$$

Since the values φ_{mm}, φ_{jj}, $\varphi_{i+2,i+2}$ are tabulated, these equations give us the factors $\varphi_{i+2,\,j}$ and $\varphi_{j,i+2}$. According to the distributive property,

$$\varphi_{i+2,\,j} = \varphi_{i+2,\,i} + \varphi_{i+2,\,i+1}. \quad (7)$$

Since all the $\varphi_{i+2,i+1}$ are already known, from (7) we can find the factors $\varphi_{i+2,i}$, i.e., the factors for nonadjacent strips. If we successively increase the number of adjacent strips combined by indices j and m, then, from formulas similar to (5)–(7), we obtain all the elements of matrix (1). All intermediate values should be arranged in tables like (2) for convenience of programming.

It may be seen from the foregoing that a total of 1/2n(n + 1) values of φ_{mm} is required. According to the above conditions, the quantity φ_{mm} does not depend on the location around the perimeter of a strip or system of strips. This allows one to limit the table of φ_{mm} to the interval $0 \le \gamma_0 \le \pi/2$ of the argument, thereby simplifying the calculation of φ_{mm}. All the φ_{mm} with $\gamma_0 > \pi/2$ are determined algebraically. Let p and q be two strips constituting a closed shell, where $\gamma_0 \le \pi/2$ for φ_{pp}; therefore φ_{pp} is taken from the table of φ_{mm}. From the meaning of the factors,

$$\varphi_{pp} + \varphi_{pq} = \xi,$$
$$\varphi_{qp} + \varphi_{qq} = \xi. \quad (8)$$

From the reciprocal relation

$$F_p\varphi_{pq} = F_q\varphi_{qp}. \quad (9)$$

Simultaneous solution of (8) and (9) gives the value of φ_{qq} (with $\gamma_0 > \pi/2$).

We now turn to tabulation of the basic factors. The general formula for φ_{mm} is obtained in a process of approximation to the integral equation of radiative energy transfer by a system of algebraic equations and has the form

$$\varphi_{mm} = \frac{1}{\Omega F_m}\int_{F_m} dF_m \int_{F_m} \exp\left(-\int_l k(l)\,dl\right)\frac{f(\Theta)\cos\Theta\, dF_m}{l^2}. \quad (10)$$

From the set of functions $f(\Theta)$ and $k(l)$ we have chosen the simplest practical forms: $f(\Theta) = \cos\Theta$ (diffuse radiation), $k(l) = k = \text{const}$. Then $\Omega = \pi$. By substituting

$$\frac{\cos\Theta\, dF_m}{l^2} = d\omega = \sin\delta\, d\delta\, d\alpha, \quad \cos\Theta = \cos\alpha\sin\delta,$$

$$l = \sqrt{H^2 + D^2\cos^2\alpha},$$

$$\frac{H}{D} = h, \; kD = \Delta, \quad \sin\delta = \frac{D}{l}\cos\alpha,$$

$$d\delta = -\frac{D\cos\alpha}{l^2}\,dH,$$

$$\frac{1}{F_m}\int_{F_m} \ldots\, dF_m = \frac{1}{2\gamma_0}\int_{\gamma} \ldots\, d\gamma, \quad d\gamma = -2d\beta$$

Table 2
Local Geometrical Factors for a Closed Cylindrical Shell ξ (s_2)

Δ	ξ	Δ	ξ	Δ	ξ	Δ	ξ
0	1.0000	0.9	0.4395	1.8	0.2150	5.5	0.0275
0.1	0.9061	1.0	0.4040	1.9	0.1997	6.0	0.0228
0.2	0.8230	1.1	0.3719	2.0	0.1857	6.5	0.0192
0.3	0.7490	1.2	0.3428	2.5	0.1313	7.0	0.0164
0.4	0.6828	1.3	0.3162	3.0	0.0953	7.5	0.0141
0.5	0.6234	1.4	0.2920	3.5	0.0710	8.0	0.0123
0.6	0.5701	1.5	0.2701	4.0	0.0542	8.5	0.0110
0.7	0.5221	1.6	0.2500	4.5	0.0423	9.0	0.0095
0.8	0.4787	1.7	0.2317	5.0	0.0338	10.0	0.0078

in place of (10) we obtain

$$\varphi_{mm} = \frac{1}{\pi\gamma_0} \int\limits_{\pi/2-\gamma_0}^{\pi/2} d\beta \int\limits_0^{\infty} dh \int\limits_{\beta}^{\pi/2} \exp(-\Delta \sqrt{h^2 + \cos^2\alpha}) \times \frac{\cos^4\alpha \, d\alpha}{(h^2 + \cos^2\alpha)^2}. \tag{11}$$

With $\Delta = 0$, $\varphi_{mm} = 1 - \sin\gamma_0/\gamma_0$.

Table 1 gives the factors φ_{mm} as calculated from (11): a Gauss quadrature with seven ordinates was used. The error is estimated to be < 1%.

For completeness Table 2, which is borrowed from Mikk [2], gives the values ξ. Mikk also gave an approximate expression for the local geometrical factors for a non-closed cylindrical strip and for a point at the edge of a strip

$$\frac{1}{2}\{s_2 - \sin\beta\,[M + \sin^2\beta\,(s_2 - M)]\}. \tag{12}$$

From the local factor (12) it is not difficult to go to an approximate formula for the mean angle factor for the strip itself:

$$\varphi_{mm} = s_2 - \frac{1}{\gamma_0}\left[M\sin\gamma_0 + (s_2 - M)\left(\sin\gamma_0 - \frac{\sin^3\gamma_0}{3}\right)\right]. \tag{13}$$

The discrepancy between our values of φ and those computed from (10), $\varphi_{(10)}$, may be expressed by the ratio $(\varphi - \varphi_{(10)})/\varphi$. The discrepancy increases with increase of Δ and with decrease of γ_0 from 0 at $\Delta = 0$ to tens of percent at $\Delta > 2$; it may be explained on the basis of the error in the local factor in (12).

NOTATION

φ is the mean generalized geometrical factor; ξ is the local generalized geometrical factor for a closed infinite cylindrical surface; F is the surface area; l is the length of ray between elements dF_m; D and H are the diameter and length of cylinder; k is the ray attenuation coefficient; Ω is the equivalent solid angle [1] determined from the condition $\varphi_{mm} = 1$ at $\Delta = 0$ for a closed surface; $f(\Theta)$ is the indicatrix of the specific radiation intensity (a quantity analogous to the specific luminous intensity [1]); Θ is the angle between the normal to an element of surface and the ray linking the surface elements; α and β are the angles in a section of the surface; α is the angle between the radius and the chord subtending the projections of dF_m on the section; β is the angle between the radius and the chord subtending the projection of element dF_m on the section and the edge of the surface; δ is the acute angle between ray l and the generator of the cylinder; γ_0 is the half subtended angle for the arc of a section of the surface; $\Delta = kD$; s_2 and M are the generalized local geometrical factors in the notation of [2] ($s_2 \equiv \xi$ and is shown in Table 2; M is the local factor for an infinite cylinder with a semicircular cross section for a point in the middle of the plane part of the surface; a table of values of M is given in [2]); n is the number of strips. Subscripts 1, 2, ..., i, ..., n, p, q, j, m indicate surfaces composed of one or more adjacent strips—zones.

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